

# Correlation between balance energy and transition energy for symmetric colliding nuclei.

Rajni and Suneel Kumar\*

*School of Physics and Materials Science,*

*Thapar University,*

*Patiala-147004, Punjab (India)*

Rajeev K. Puri

*Department of Physics, Panjab University,*

*Chandigarh-160014 (India)*

(Dated: January 26, 2013)

## Abstract

We study the correlation between balance energy and transition energy of fragment in heavy-ion collisions for different systems at incident energies between 40 and 1200 MeV/nucleon using an isospin-dependent quantum molecular dynamics model. With increasing incident energy, the elliptic flow shows a transition from positive (in-plane) to negative (out-of-plane) flow. This transition energy is found to depend on the size of fragments, composite mass of reacting system, and the impact parameter of reaction. It has been observed that reduced cross-section can explain the experimental data. There is a correlation between transition energy and balance energy as their difference decreases with increase in the total mass of colliding nuclei.

PACS numbers: 25.70.-z, 25.75.Ld

---

\*Electronic address: suneel.kumar@thapar.edu

## I. INTRODUCTION

The elliptic flow is a measure that quantifies the azimuthal anisotropy of the momentum distribution. Specifically, we fit the azimuthal distribution of nucleons about reaction plane with a Fourier expansion of the form:

$$\frac{dN}{d\phi} = v_0(1 + 2v_1\cos\phi + 2v_2\cos2\phi + \dots). \quad (1)$$

where,  $v_0$  is for normalization only,  $v_1$  characterizes the directed in-plane flow, while  $v_2 > 0$  indicates in-plane enhancement,  $v_2 < 0$  characterizes the squeeze-out perpendicular to the reaction plane, and  $v_2 = 0$  shows an isotropic distribution of nucleon momentum in the transverse plane. Hence, the ellipticity coefficient  $v_2$  depends on the in-plane and out-of-plane flow amplitudes. The elliptical flow parameters  $\langle\cos2\phi\rangle$  at energies from tens to hundreds of MeV per nucleon are determined by the complex interplay among expansion, rotation, and the shadowing of spectators. Both the mean field and two body collision parts play important roles in this energy region. The mean field play a dominant role at low energies, and then gradually the two body collision becomes dominant with increase in energy. Two colliding nuclei create a stopped overlap region. At higher bombarding energies ( $E_{lab} \geq 1\text{GeV/nucleon}$ ) the spectator leave interaction zone rapidly. The remaining interaction zone expands almost freely, where the surface is such that in-plane emission is preferred. It is therefore also the interplay between the timescales of passing time of spectators and expansion time of the dense, stopped interaction zone which determines the time-integrated elliptic flow signal. Experimentally observed out-of-plane emission, termed as squeeze was first observed by at SATURNE (France) by the DIOGENE Collaboration [1]. Plastic Ball group at the BEVALAC in BERKLEY were the first one to quantify the squeeze out in symmetric systems [2]. Recently, elliptic flow has been measured at relativistic heavy ion collider (RHIC) in Au+Au collision at  $\sqrt{s} = 130\text{GeV/nucleon}$  [3]. At top AGS and SPS energies, elliptic flow is inferred to be a relative enhancement of emission in the plane of the reaction. Elliptic flow is developed mostly in the first several fm/c (of the order of the size of nuclei) after the collision and thus provides information about the early-time thermalization achieved in the collision [4]. A large elliptic flow of all charged particles near midrapidity was reported by STAR Collaboration. The FOPI, INDRA and PLASTIC BALL Collaboration [5, 6] are actively involved in measuring the

excitation function of elliptic flow from Fermi energies to relativistic energies. Interestingly, at intermediate energies ( $E_{lab} \approx 100 \text{ MeV/nucleon}$ ) change from in-plane emission (rotation like behaviour) to squeeze-out is predicted [7, 8] whereas at relativistic energies ( $E_{lab} \approx 5 \text{ GeV/nucleon}$ ) the opposite change from the squeeze-out to in-plane enhancement is observed. Elliptic flow requires reinteractions within the produced matter as a mechanism for transferring the initial spatial deformation of the reaction zone in noncentral collision onto momentum space. It is thus plausible to expect that the largest elliptic flow signal is produced in the hydrodynamic limit and an almost linear increase in its value with the particle transverse momentum below 1.5 GeV/c. In the hybrid model of combining the hydrodynamic model with the RQMD transport model [9] and choosing certain effective equation of state, it is possible to obtain an elliptic flow that is comparable to the measured ones in heavy-ion collisions at both SPS and RHIC energies [10]. The experimental result shows that elliptic flow first increases with particle transverse momentum and then levels off. The dependence of elliptic flow on both the charged particle multiplicity [11, 12] and the particle pseudorapidity [12] have also been measured. A complete study of excitation function of transverse momentum and energy dependence of elliptic flow in the entire energy region can provide useful information about nucleon-nucleon interaction related to nuclear equation of state. In literature, many attempts have already been made with hard equation of state with free N-N cross-section and soft EOS with reduced nucleon-nucleon cross-section with and without momentum dependent interactions and also tried to explore different aspects of directed sideward flow. This study is in continuation with our previous study [13], in which we have shown that experimental balance energies can be explained well with reduced isospin dependent NN cross-section with hard equation of state. In the present study our aim is to pin down the relation between balance energy and transition energy. Is there any relation between these two energies. Whether there is any mass dependence or not. For the present study, the isospin dependent quantum molecular Dynamics (IQMD) model is used to generate the phase space of nucleons [14].

## II. RESULTS AND DISCUSSION

We study the elliptic flow using a stiff equation of state along with isospin-dependent reduced cross-sections ( $\sigma = 0.9 \sigma_{NN}$ ), by simulating various reactions. The time evolution of the reaction is followed upto 200 fm/c. This is the time at which flow saturates for lighter as well as for heavier systems. For this study, the reactions of  $^{40}\text{Ar}_{18} + ^{45}\text{Sc}_{21}$  ( $\hat{b} = 0.4$ ,  $L=0.5L$ ),  $^{93}\text{Nb}_{41} + ^{93}\text{Nb}_{41}$  ( $\hat{b} = 0.3$ ,  $L=0.7L$ ),  $^{139}\text{La}_{57} + ^{139}\text{La}_{57}$  ( $\hat{b} = 0.3$ ,  $L=0.8L$ ), and  $^{197}\text{Au}_{79} + ^{197}\text{Au}_{79}$  ( $b=2.5\text{fm}$ ,  $L=L$ ) are simulated, where  $L$  is the Gaussian width. As mentioned in Ref. [14], in IQMD the value of Gaussian width  $L$  depends on the size of the system. For Au nuclei  $L=8.66 \text{ fm}^2$  and for Ca nuclei  $L=4.33 \text{ fm}^2$ .  $\hat{b}$  is the scaled impact parameter is defined as  $\hat{b} = \frac{b}{b_{max}}$  (where  $b$  is particular impact parameter in Fermi(fm) and  $b_{max} = 1.12(A_T^{1/3} + A_P^{1/3})$ ),  $A_T$  and  $A_P$  is the mass of target and projectile respectively. The choice of impact parameter is guided by the experimentally extracted information [15–17]. These reaction have been performed at their corresponding balance energies. The above reactions were simulated between 40 and 1200 MeV/nucleon using the hard equation of state along with isospin-dependent reduced cross-sections. The phase space generated by the IQMD model has been analyzed using the minimum spanning tree (MST) [18] method. The MST method binds two nucleons in a fragment if their distance is less than 4 fm. In recent years, several improvements have also been suggested. One of the improvements is to also imply momentum cut of the order of Fermi momentum. This method is dubbed as MSTM method [19]. The entire calculations are performed at  $t = 200 \text{ fm/c}$  i.e. (Saturation time).

The elliptical flow is defined as the average difference between the square of the  $x$  and  $y$  components of the particle's transverse momentum. Mathematically, it can be written as

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle, \quad (2)$$

where  $p_x$  and  $p_y$  are the  $x$  and  $y$  components of the momentum. The  $p_x$  is in the reaction plane, while,  $p_y$  is perpendicular to the reaction plane.

In the Fig.1, we display the transverse momentum dependence of elliptical flow for the free particles, light charged particles. A Gaussian-type behavior is observed in all cases. Note that this elliptical flow is integrated over entire rapidity range. It is also evident from the figure that the peaks of the Gaussian shifts toward lower values of  $P_t$  for heavier fragments.

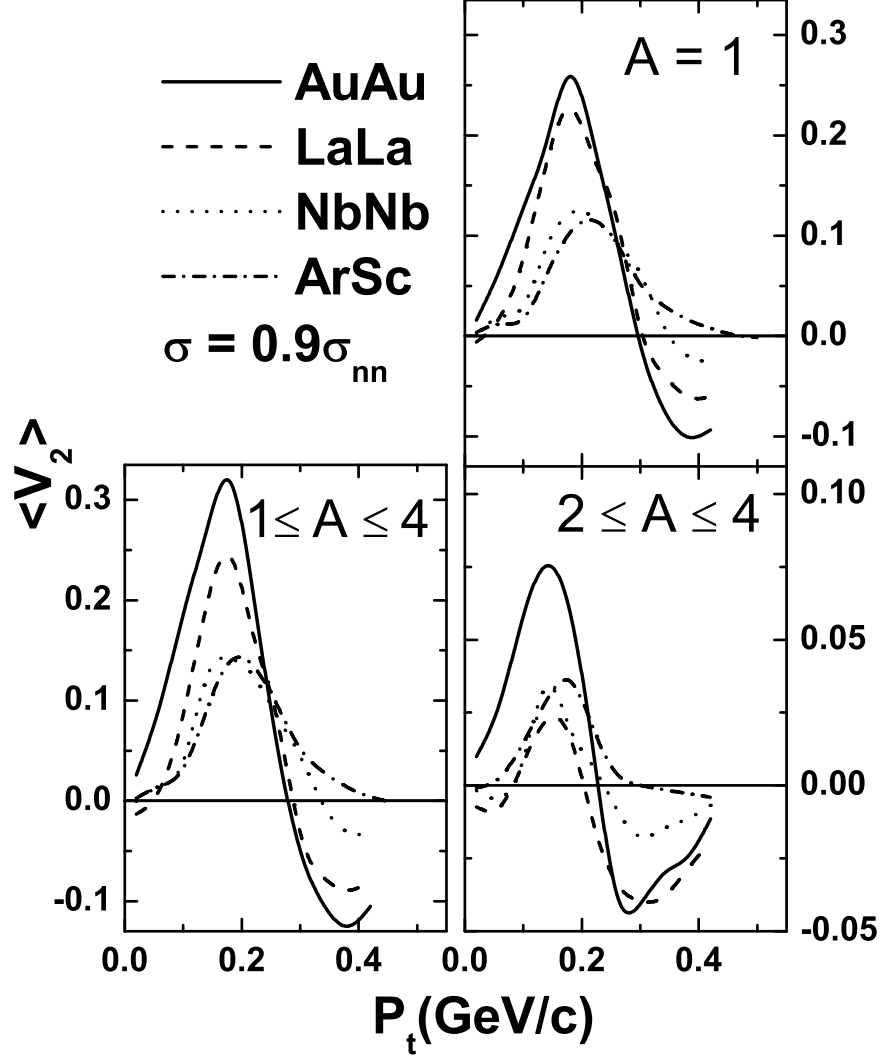


FIG. 1: Transverse momentum dependence of the elliptical flow at  $E = 200$  MeV/nucleon. The different lines in the figure show the variation with different system mass and different panels show the fragments of different mass range.

This is due to the fact that the free and light charged particles feel the mean field directly, while heavy fragments have weaker sensitivity [20]. In the Fig.2, we display the system size dependence of the transverse momentum at which  $v_2$  becomes zero for different system and different fragments. The value of transverse momentum decreases with system mass because

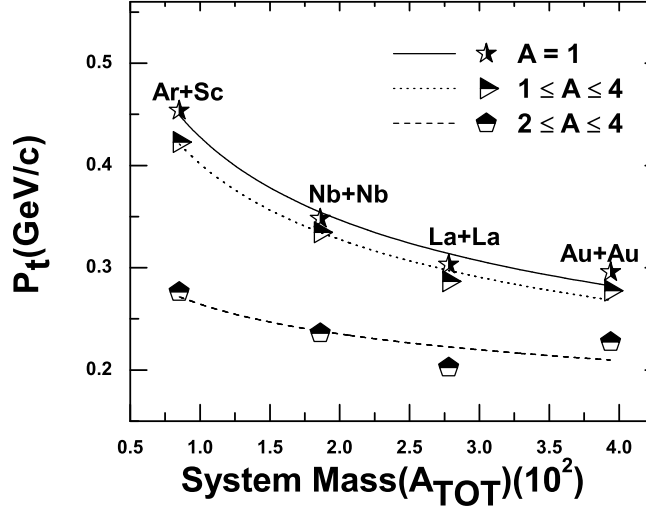


FIG. 2: Transverse momentum dependence ( $P_t$ ) for elliptic flow as a function of the combined system mass for different systems with fragments of different mass range

in heavier systems there are more Coulomb repulsions than that of lighter systems. This dependence is again fitted with the power law of the kind

$$E_{Trans} = C(A_{tot}^{-\tau}) \quad (3)$$

The value of  $\tau$  is found to increase with increase in fragment mass range. As in case of  $A = 1$  it is  $-0.30 \pm 0.03$ , for  $1 \leq A \leq 4$  it is  $-0.29 \pm 0.02$ , for  $2 \leq A \leq 4$  it is  $-0.17 \pm 0.07$ . In the fig.3, we display the variation of the elliptic flow  $v_2$  for free nucleon, light charge particle (LCP's) over midrapidity region as a function of energies. The free particles and LCP's, which originate from the participant zone, show a systematic behavior with the beam energy and with the composite mass of the system as well as with the fragments of different mass range. The elliptical flow for these particles is found to become more negative with the increase in the composite mass of system and with the increase in the beam energy as well as with the fragments of different mass range. The heavier the system, the greater the Coulomb repulsion and more negative is the elliptical flow.

The elliptical flow is found to show a transition from in-plane to out-of-plane at a certain beam energy known as transition energy for mid-rapidity region. This is due to the change in the rotational behavior into expansion with increase in the incident energy.

In the fig.4, we display the system size dependence of the difference of transition

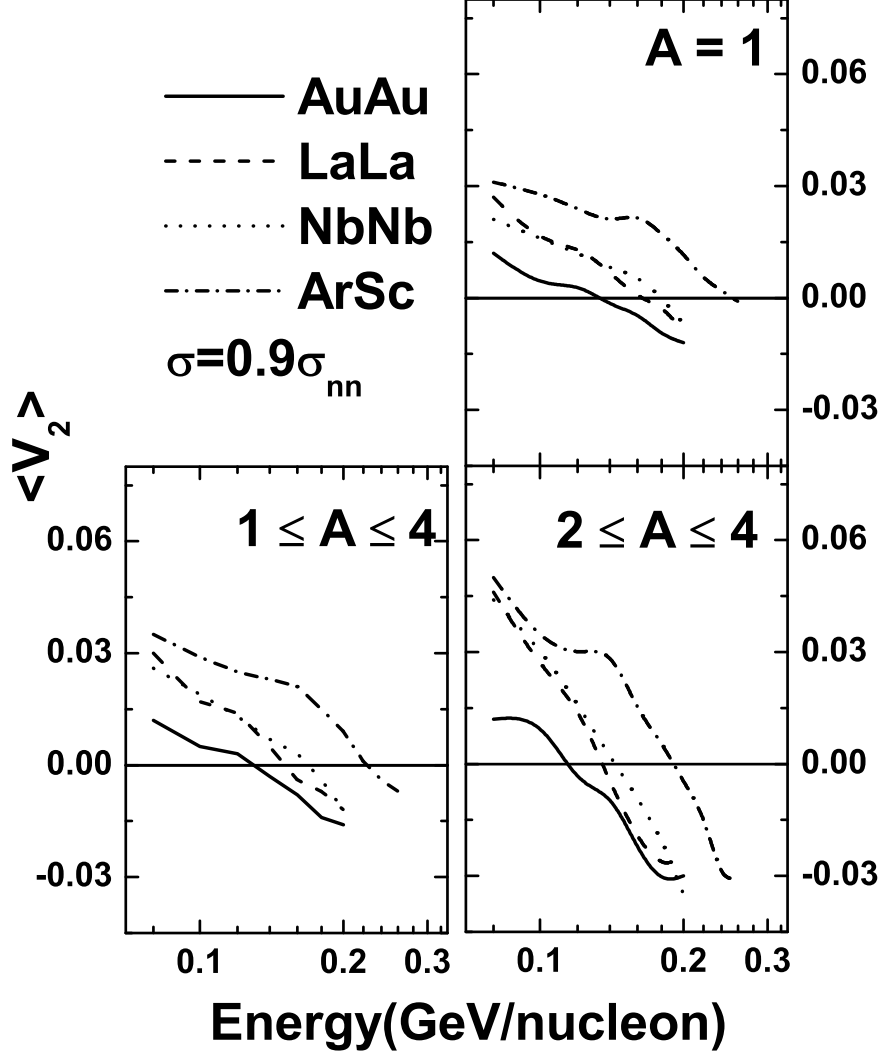


FIG. 3: Variation of the elliptic flow, with beam energy at  $|y| = |\frac{y_{c.m.}}{y_{beam}}| \leq 0.1$  for different reactions.

energy( $\Delta E(\%) = \frac{E_t - E_b}{E_b} \times 100$ ) extracted from the fig.3 for different fragments and balance energy( $\sigma = 0.9\sigma_{NN}$ ) studied in ref. [13]. For the fragment of low mass range this difference show slight decrease as we increase the system mass then other effect comes into play (i.e. expansion of participant and shadowing of spectator matter) whereas for light charge particle's constant line is obtained which means the additional effect is independent of system mass. It shows that transition energy and balance energy are closer in heavy

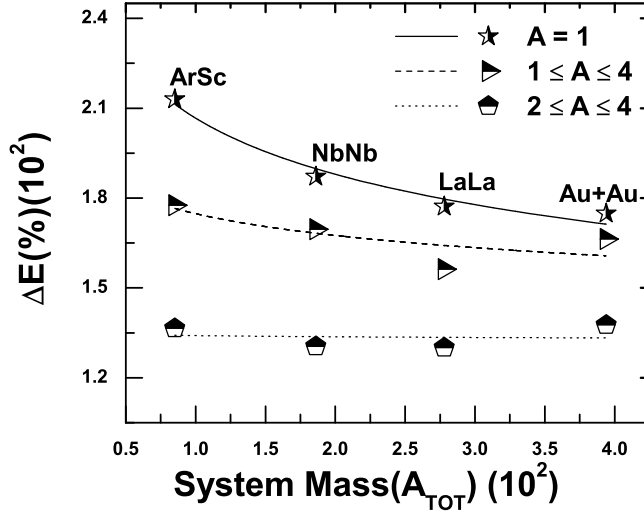


FIG. 4: Difference of transition energy and balance energy as a function of mass of system.

mass system as compared to lighter systems. This happens due to increase of neutron, the number of collision increases and hence leads to decrease in transition energy. This dependence is again fitted with the power law. The value of  $\tau$  is found to increase with increase in fragment mass range. As in case of  $A = 1$  it is  $-0.14 \pm 0.017$ , for  $1 \leq A \leq 4$  it is  $-0.061 \pm 0.036$ , for  $2 \leq A \leq 4$  it is  $-0.0043 \pm 0.031$ .

In the fig.5, we show  $v_2$  at midrapidity  $|y| = |\frac{y_{c.m.}}{y_{beam}}| \leq 0.1$  for  $Z=2$  as a function of incident energy. The rapidity cut is in accordance with the experimental findings. The theoretical results are compared with the experimental data extracted by INDRA, FOPI and PLASTIC BALL collaborations[5, 6]. With the increase in the incident energy, elliptical flow  $v_2$  changes from positive to negative values exhibiting a transition from the in-plane to out-of-plane emission of nucleons. This is because of the fact that the mean field, which contributes to the formation of a rotating compound system, becomes less important and the collective expansion process based on the nucleon-nucleon scattering starts to be predominant. The maximal negative value of  $v_2$  is obtained around  $E = 500$  MeV/nucleon with reduced isospin dependent cross-section. This out-of-plane emission decreases again towards the higher incident energies. This happens due to faster movement of the spectator matter after  $v_2$  reaches the maximal negative value [6]. This trend is in agreement with experimental findings. A close agreement with data is obtained in the presence of hard equation of state and with reduced isospin dependent cross-section for



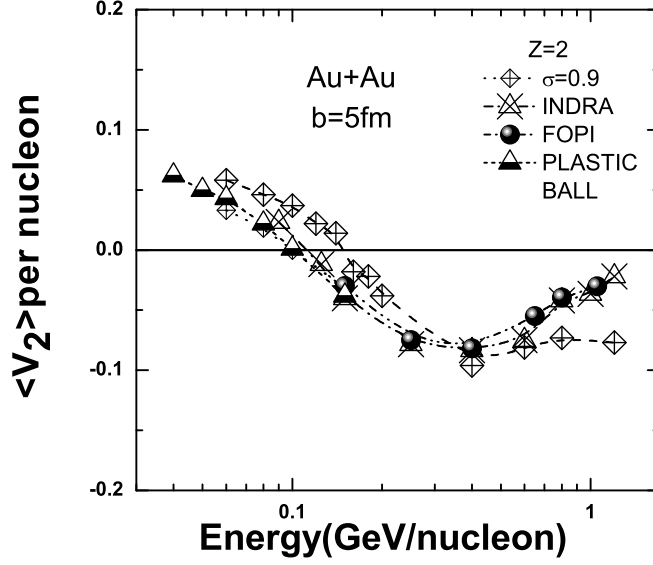


FIG. 5: Energy dependence of the elliptic flow for Au+Au systems and its comparison with the experimental data.

Z=2 particles. Similar results and trends have also been reported by Zhang et.al. in their recent communication [21].

### III. CONCLUSION

By using the IQMD model, we have studied correlation between transition energy and balance energy. We have investigated the elliptical flow of fragments for different reacting systems at incident energies between 40 and 1200 MeV/nucleon using isospin-dependent quantum molecular dynamics (IQMD) model. The elliptical flow is found to show a transition from in-plane to out-of-plane at a certain beam energy in mid-rapidity region. Our calculation with a stiff equation of state and reduced isospin dependent nucleon-nucleon cross-section ( $\sigma = 0.9\sigma_{NN}$ ) is in good agreement with the experimental findings. The difference between balance and transition energy decreases with increase in the composite mass of colliding nuclei. This tells us that due to increase of neutron to colliding nuclei, the difference between two energies decreases.

## Acknowledgments

Authors are thankful to Prof. J. Aichelin for discussion during his visit to Panjab University Chandigarh under Indo French joint International project.

---

- [1] J. Gosset *et al.*, Phys. Rev. C **16**, 629 (1977); J. Gosset *et al.*, Phys. Rev. Lett. **62**, 1251 (1989).
- [2] R. Renfordt *et al.*, Phys. Rev. Lett. **53**, 763 (1984).
- [3] Zi-Wei Lin and C. M. Ko, Phys. Rev. C **65**, 034904 (2002).
- [4] R. J. Snellings, STAR Collaboration, Nucl. Phys. A **698**, 193 (2002).
- [5] J. Lukasik, G. Auger, M. L. Begemann-Blaich *et al.*, Phys. Lett. B **608**, 223 (2005).
- [6] A. Andronic *et al.*, Nucl. Phys. A **679**, 765 (2001); Phys. Lett. B **612**, 173 (2005).
- [7] S. Soff *et al.*, Phys. Rev. C **51**, 3320 (1995).
- [8] P. Crochet *et al.*, (FOPI Collab.), Nucl. Phys. A **624**, 755 (1997).
- [9] H. Sorge, Phys. Rev. C **52**, 3291 (1995).
- [10] D. Teaney, J. Lauret, and E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001); Nucl. Phys. A **698**, 479 (2002).
- [11] K. H. Ackermann *et al.*, STAR Collaboration, Phys. Rev. Lett. **86**, 402 (2001).
- [12] C. Roland *et al.*, PHOBOS Collaboration, Nucl. Phys. A **698**, 54 (2002); I. C. Park *et al.*, PHOBOS Collaboration, *ibid.* **698**, 564 (2002).
- [13] S. Kumar, Rajni, S. Kumar, Phys. Rev. C **82**, 024610 (2010), S. Kumar and S. Kumar, Pramana J. of Physics **74**, 731 (2010).
- [14] C. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stöcker and W. Greiner, Eur. Phys. J. A **1**, 151 (1998); S. Kumar, S. Kumar and R. K. Puri, Phys. Rev. C **81**, 014601 (2010); S. Kumar and S. Kumar, Chin. Phys. Lett. **27**, 062504 (2010).
- [15] G. D. Westfall *et al.*, Phys. Rev. Lett. **71**, 1986 (1993); D. J. Magestro, W. Bauer, O. Bjarki, J. D. Crispin, M. L. Miller, M. B. Tonjes, A. M. Vander Molen, G. D. Westfall, R. Pak and E. Norbeck, Phys. Rev. C **61**, 021602(R) (2000); D. J. Magestro, W. Bauer and G. D. Westfall,

- Phys. Rev. C **62**, 041603(R) (2000).
- [16] R. Pak *et al.*, Phys. Rev. C **53**, R1469 (1996); Phys. Rev. Lett. **78**, 1022 (1997); *ibid.* **78**, 1026 (1997).
  - [17] D. Cussol *et al.*, Phys. Rev. C **65**, 044604 (2002). 314,1 96
  - [18] J. Aichelin, Phys. Rep. **202**, 233 (1991).
  - [19] J. Singh and R. K. Puri, Phys. Rev. C **62**, 054602 (2000).
  - [20] T. Z. Yan *et al.*, Chin. Phys. **16**, 2676 (2007).
  - [21] Y. Zhang and Z. Li, Phys. Rev. C **74**, 014602 (2006).